MAXIMUM BRILLIANCY OF VENUS.

By Arthur B. Turner.

The brightness of a star, as far as we can tell, is due to the fact that it is a glowing mass in space, sending out waves of light in all directions, a small fraction of which are gathered by the lens of the eye, or the eye aided by the telescope, and give us the sense of its existence. A star may be intrinsically bright or it may be bright because it is close to our solar system.

Astronomers designate the brightness of a star by an algebraic number, which is called its magnitude. The larger the number the fainter the star. On such a scale *Polaris* is of magnitude 2.2, *Aldebaran* 1.0, Sirius —1.4, and the Sun —26.3. A star of magnitude 1 is approximately 2½ times as bright as a star of magnitude 2, which is the common ratio of brightness between any two consecutive magnitudes on down the scale.

The observations made at different times on the same star, however, often show that there is a change in the magnitude, generally very slight, altho sometimes quite large. Over four thousand such stars have already been observed. Many of these change their magnitude quite irregularly; others are periodic, repeating their maximum and minimum brightness at regular intervals, ranging from 3 hours 12 minutes to 610 days. Why the stars change in magnitude we do not know, except in those cases where the spectroscope gives additional information, showing that the star is not single but double, each star periodically partially eclipsing the other as they revolve around their common center of gravity.

Passing over the comets and meteors, the former of which, altho self-luminous, have their brightness violently altered on their approach to the Sun, and the latter of which, tho cold as outside space, become suddenly luminous from friction when they rush into the Earth's atmosphere, we come to a class of star-like bodies in the sky most of which shine entirely by reflected light—the planets and their satellites. These bodies will vary in brightness from the following causes: (a) the dis-

tance from the source of light, (b) the distance from the observer, (c) the amount of illuminated surface visible (i. e. the phase), (d) the reflecting power of the body, called its albedo, and (e) possibly the rotation of the planet or satellite on its axis, since all portions of their surfaces may not have the same reflecting power.

Next to the Moon, which is very conspicuous in her monthly changes of brightness with accompanying phases, we find the planet *Venus* changing in magnitude, but curiously reaching its maximum brightness in the crescent phase. Since the orbit of *Venus* is very nearly circular and but slightly inclined to the ecliptic, a rather simple mathematical demonstration of the above fact is possible. The principle is also applicable in the case of the other planets, but with a lesser degree of accuracy.

Let us assume, then, that the orbits of the Earth and *Venus* are two concentric circles, with the Sun as the common center. Let r be the radius of the orbit of *Venus*, ρ the distance between the Earth and the planet at any given instant, and ψ the angle at *Venus* between the Sun and Earth (see Figure 3), then the intensity (I) of the light reaching the Earth is given by the equation—

$$\mathrm{I} = K \frac{(1 + \cos\psi)\,\tfrac{1}{2}\pi a^2}{\rho^2 r^2}$$

K is a constant and a is radius of *Venus*. The numerator $\tfrac{1}{2}\pi a^2\,(1 + \cos\psi)$ is the area of the surface of the planet that is visible [for example, when $\psi = 0^0$, $\cos\psi = 1$ and we get πa^2 as full phase; again when $\psi = 90^\circ$, $\cos\psi = 0$, and we get $\tfrac{1}{2}\,\pi a^2$, or half phase; also when $\psi = 180^\circ$, $\cos\psi = -1$, and we get 0 area or new phase]. Since light varies inversely as the square of the distance both of the observer from *Venus* and of *Venus* from the Sun, we have the product of the two squares as the denominator. But by trigonometry $\Delta^2 = \rho^2 + r^2 - 2r\rho\cos\psi$, whence—

$$\mathrm{I} = K \frac{[(r + \rho)^2 - \Delta^2]}{r^3\rho^3}\,\tfrac{1}{2}\pi a^2$$

It is evident that for a given value of r (i. e. for each planet) there will be generally one value of ρ which will make I, the planet's brilliancy, a maximum. From this value of ρ we can

easily calculate the triangle for angle ψ and hence find the phase of the planet, when it has maximum brilliancy as viewed from the Earth. This value of ρ can be obtained approximately by the method of "cut and try" (i. e. trying values of ρ until we get the largest value of ρ possible for a given value of r). The differential calculus, however, enables us to solve the problem directly, and we easily find that ρ must be a root of the quadratic equation $\rho^2 + 4r\rho + 3r^2 = 3\Delta^2$ to give a maxi-

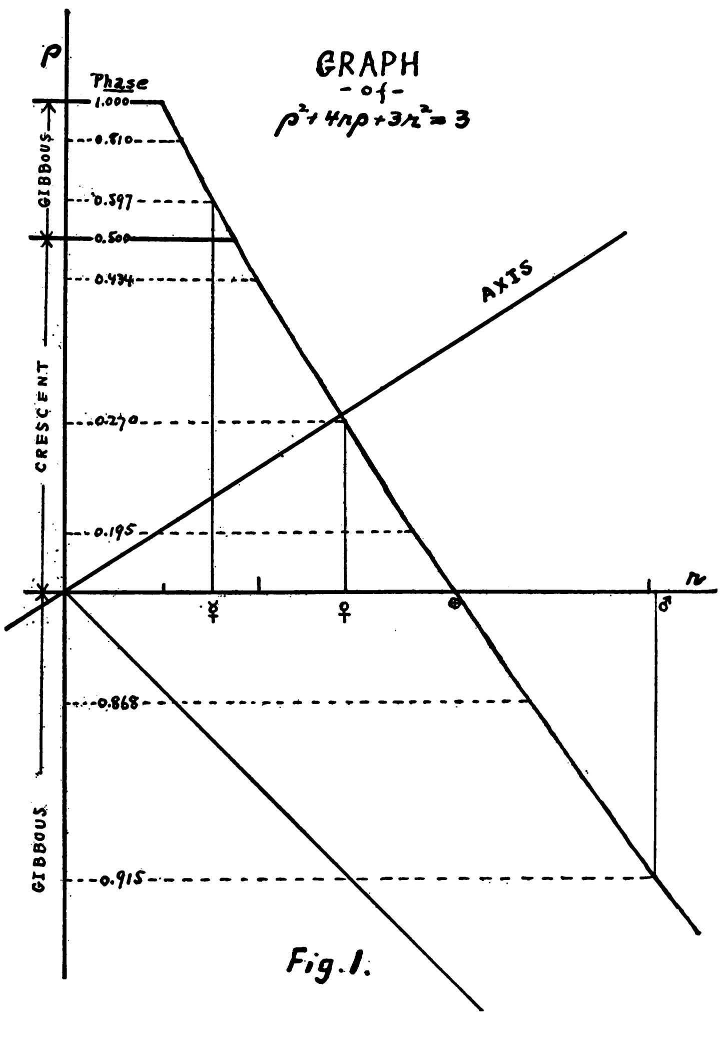

Fig. 1.

mum value of I; each value of r giving a different value of ρ. That is, in general, no two planets will be in exactly the same phase or distance from the Earth when at maximum brilliancy.

Figure 1 is a graphical representation of this equation, where Δ is taken as the unit of distance. Only a portion of one branch of the curve, an hyperbola, is shown as it is sufficient to illustrate the argument. The axis of the curve is shown and also one of the tangents at infinity—an asymptote. For nine values of ρ the phase has been calculated, and is indicated on the horizontal dotted lines by the decimal of the disk illuminated. For example: (1) *Mercury* (☿), which is at a distance $r = 0.387$ from the Sun, must be at a distance $\rho = 1.001$

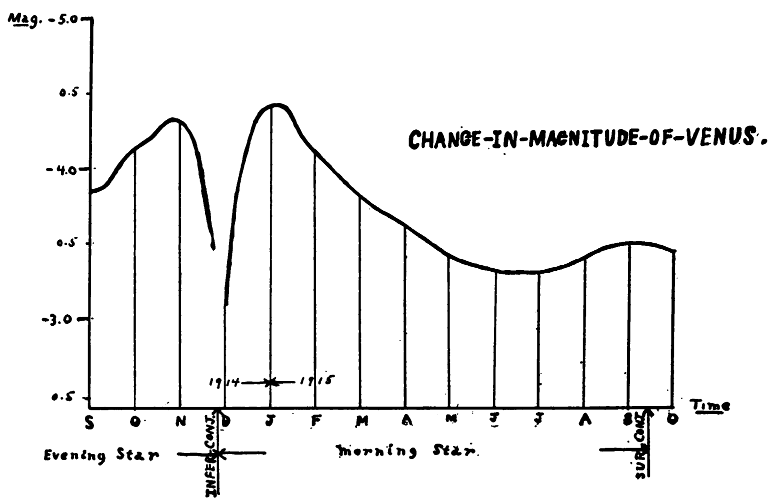

Fig. 2.

from the Earth, when its brightness is a maximum. The planet will then be 0.597 illuminated, and will appear in the gibbous phase in the telescope. (2) *Venus* (♀) has $r = 0.723$, which gives $\rho = 0.431$, and 0.270 of the disk is illuminated and the planet will be seen as a crescent about like the Moon when it is between four and five days old. (3) *Mars* (♂) has $r = 1.524$, $\rho = -0.741$ and 0.915 of the disk will be illuminated, almost full phase when its brightness is at maximum. The diagram also shows that for planets with r between 0.25 and

0.44 the phase is gibbous, for r between 0.44 and 1.00 it is a crescent, and for values of r greater than 1 the phase is again gibbous; that is, the farther the planet is from the Sun the closer it will be to opposition when at maximum brightness.

In order to show the rapid changes in the magnitude of *Venus,* Figure 2 was drawn from data taken from the *American Ephemeris.* Beginning with September 1, 1914, numerous dates with corresponding magnitudes have been plotted up to October 1, 1915. Thru these points a curve was drawn. The time axis was taken at magnitude —2.4, which is the maximum brilliancy of *Jupiter,* for purposes

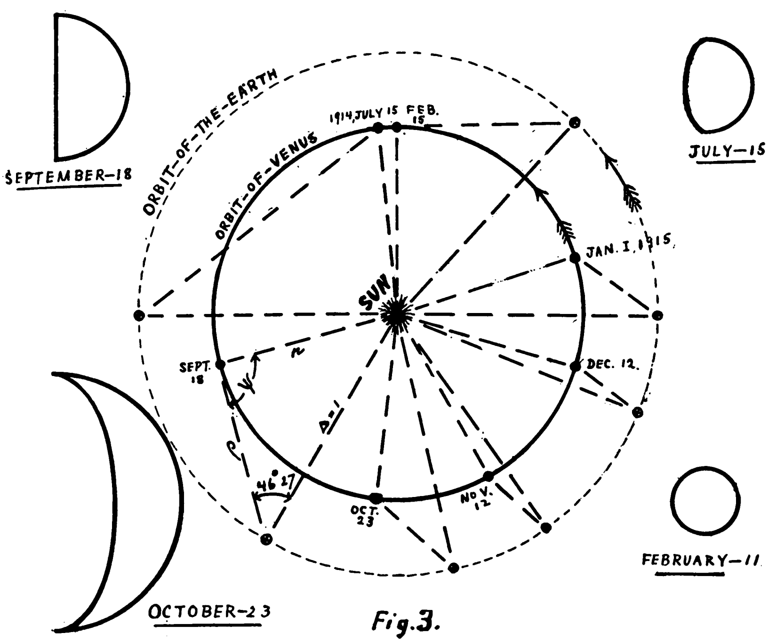

Fig. 3.

of comparison. It is noticed that *Venus* reaches a maximum (—4.4) toward the end of October, 1914, when an evening star, and again in the first part of January, 1915, when a morning star. At these two times the planet is 6¼ times as bright as the maximum of *Jupiter* and 180 times as bright as *Polaris.* The gap in the curve is during the passage thru inferior conjunction. [The phase of *Venus* and its distance

from the Earth at maximum brightness as found from Figure 1 agree very well with the more accurate values of the *American Ephemeris.*] The reflecting power of the surface of *Venus* is three times that of the surface of the Moon, and this is probably due to the fact that *Venus* is enveloped in clouds, while the Moon has practically no atmosphere whatever. In thirty-six days the planet passes from inferior conjunction to such a magnitude that it can be seen in full daylight. The maximum brightness of *Venus,* either as a morning or an evening star, is repeated every 584 days.

Figure 3 shows the relative positions of the Earth and *Venus* for seven dates from July 15, 1914, to February 15, 1915. This interval includes the two dates of maximum brilliancy. Four phases and corresponding changes in apparent diameter are also shown, the full phase of superior conjunction (February 11th) being taken as the unit. On September 18th the planet reaches maximum eastern elongation from the Sun and is just half illuminated (magnitude —4.1). [In this position the Earth, *Venus* and the Sun form a right triangle.] The sine of this angle (46° 27′), which is 0.725, gives a very good approximation to the distance of *Venus* from the Sun. On October 23d the planet is a large crescent and of maximum brightness (—4.4 magnitude). The last of November, 1914, the planet is again in line with the Sun (inferior conjunction). After this time *Venus* becomes a morning star, rising before the Sun, and then repeats the phases, reaching maximum again on January 1, 1915, and superior conjunction in September, 1915.

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